# PREDICTION OF PRESSURE DROP
# OF SLUG FLOW IN VERTICAL PIPES USING MECHANISTIC MODEL

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## ABSTRACT

Due to the variability of flow pattern of gas liquid two-phase flow and complexity of flow mechanism,it is very difficult to seek a single model which is able to predict pressure drop and fit for any flow condition. when the existing model of two-phase flow pressure drop is used to predict the pressure of the conditions of producing gas well,a large error occurs.Therefore, it is necessary,based on the experimental data of gas-water two phase flow,to research the flow mechanism and discover the regular existing in the process of fluid property changing .On the basis of the current two-phase flow pressure drop model, it is important to explore modified pressure loss model applicable for producing gas well with water,to improve predictability of the pressure drop of gas wells,and to provide the theory and technology guidence for development of gas reservoir with water.

Underbalanced drilling (UBD) has increased in recent years because of the many advantages associated with it. These include increase in the rate of penetration and reduction of lost circulation and formation damage. Drilling of deviated and horizontal wells also increased since recovery can be improved from a horizontal or a deviated well. The drilling of deviated wells using UBD method will reduce several drilling related problems such as hole cleaning and formation damage. Prediction of flow and pressure profiles while drilling underbalanced in such wells will help in designing and planning of the well. The aim of this research is to predict the pressure drop of slug flow in the certain pressure in vertical pipes using mechanistic model and to study the behavior of the flow profile in the drillstring and the annulus under UBD conditions through the use of mechanistic two phase flow models.

Mechanistic two phase flow models is been used In this research to predict the liquid hold up for phase gas- liquid slug flow which is important for the accurate calculations of the pressure drop.In particular, its evaluation is important for the vertical pipes since the liquid hold up in the slug body is the main contributor to the hydrostatic pressure drop which quite significant for the verticals flows. Further development of mechanistic models has allowed accurate prediction of wellbore pressure. Many Underbalanced Drilling operations require the use of nitrified diesel as the drilling fluid.Thus two phase flow will exist both in the drill pipe and the annulus.

# ABSTRAK

Oleh kerana kepelbagaian dalam corak aliran gas-cecair aliran dua fasa dan kerumitan mekanisme aliran, ia adalah amat sukar untuk mendapatkan model tunggal yang mampu meramalkan kejatuhan tekanan dan sesuai untuk sebarang keadaan aliran. apabila model yang sedia ada dua fasa kejatuhan tekanan aliran digunakan untuk meramalkan tekanan dengan syarat-syarat dan keadaan untuk mengeluarkan gas dengan baik, kesilapan yang besar berlaku. Oleh itu, adalah perlu berdasarkan data eksperimen gas-cecair aliran dua fasa, dengan penyelidikan mekanisme aliran dan penemuan yang sedia ada dalam proses perubahan sifat bendalir. Berdasarkan dua fasa mod kejatuhan tekanan aliran semasa, ia adalah penting untuk pengubahsuaian model kehilangan tekanan yang diguna pakai untuk telaga gas yang mengandungi air, untuk meningkatkan ketepatan ramalan penurunan tekanan telaga gas, dan untuk menyediakan teori dan teknologi untuk pembangunan takungan telaga gas yang mengandungi air.

Penggerudian Underbalanced (UBD) telah meningkat sejak kebelakangan ini kerana banyak kelebihan yang berkaitan. Ini termasuk peningkatan dalam kadar penembusan dan pengurangan kehilangan edaran dan kerosakan formasi. Penggerudian telaga terpesong dan mendatar juga meningkat kerana proses pemulihan juga boleh diperbaiki dari melintang atau menyimpang. Penggerudian telaga lencongan menggunakan kaedah UBD akan mengurangkan beberapa masalah penggerudian yang berkaitan seperti pembersihan lubang dan kerosakan formasi. Ramalan aliran dan tekanan profil semasa penggerudian underbalanced dalam telaga seumpama itu akan membantu dalam mereka bentuk dan perancangan telaga. Tujuan kajian ini adalah untuk meramalkan kejatuhan tekanan aliran lumpur dalam tekanan tertentu di dalam paip yang menegak menggunakan model mekanistik dan untuk mengkaji kelakuan profil aliran di drillstring dan anulus dalam keadaan UBD melalui penggunaan mekanistik aliran dua fasa model.

Mekanistik dua model aliran fasa telah digunakan dalam kajian ini untuk meramalkan cecair tahan untuk fasa gas-cecair di dalam aliran lumpur yang penting untuk pengiraan penurunan tekanan yang tepat. Secara khususnya, penilaian ini adalah penting bagi paip yang menegak kerana cecair tahan di dalam aliran lumpur adalah penyumbang utama kepada kejatuhan tekanan hidrostatik yang agak ketara untuk aliran menegak. Pembangunan model mekanistik telah membenarkan ramalan yang tepat tekanan lubang telaga. Banyak operasi Penggerudian Underbalanced memerlukan penggunaan diesel nitrified sebagai cecair penggerudian. maka dua aliran fasa akan wujud kedua-dua di dalam paip gerudi dan anulus.

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$\left(\frac{dP}{dL}\right)_{acc}$ Acceleration pressure gradient, (psi/ft)

$\left(\frac{dP}{dL}\right)_{el}$ Elevation pressure gradient, (psi/ft)

$\left(\frac{dP}{dL}\right)_{f}$ Frictional pressure gradient, (psi/ft)

$\left(\frac{dP}{dL}\right)_{total}$ Total pressure gradient, (psi/ft)

$A_L$ Liquid area in pipe element (m2,in2)

$A_n$ Bit nozzle area (m2,in2)

$A_p$ Pipe element area, (m2,in2)

$C_1$ Velocity profile coefficient for slug flow

$C_0$ Velocity profile coefficient for bubbly flow

$D_e$ De Equivalent pipe diameter, (m/in)

$D_{ep}$ Equi-periphery diameter, (m/in)

$D_h$ Hydraulic diameter, (m/in)

$E_a$ Absolute average relative error

$f_F$ Fanning friction factor

$f_i$ Interfacial shear friction factor for annular flow

$f_M$ Mixture friction factor

$f_p$, $F_{CA}$, Geometry parameters in calculating fanning friction factor for bubbly flow

$f_{SC}$ Superficial core friction factor

$H_L$ Liquid holdup

$H_L^n$ Liquid holdup with swarm effect

$H_{L_{LS}}$ Liquid Holdup in liquid slug zone

$H_{L_{SU}}$ Liquid Holdup in a slug unit

$H_{L_{TB}}$ Liquid Holdup in Taylor bubble in a slug flow

ID Inner diameter (m,in)

$L_{L_{LS}}$ Liquid length in liquid slug zone

$L_{L_{TB}}$ Length of slug unit

$L_{L_{TB}}$ Slug length in Taylor bubble in a slug

Mg Gas molecular weight

$N_{Re}$ Reynolds Number

$N_{Re,M}$ Mixture Reynolds number

$N_{Re,SG}$ Superficial gas Reynolds number

$N_{Re,SL}$ Superficial liquid Reynolds number

OD Outer diameter (m,in)

$P_{bh}$ Bottom hole pressure (Pa,psi)

$P_{calc}$ Calculated Pressure (Pa,psi)

$P_{meas}$ Measured Pressure (Pa,psi)

$P_{up}$ Upstream pessure (Pa,psi)

$q_G$ Gas flow rate (scf/m)

$q_L$ Liquid flow rate, (m3/s, gpm)

R Universal Gas constant = 10.731 psia.ft3/lbm.mol.°R

T Temperature (°K,°R)

$w_g$ Gas weighing factor

Z Gas compressibility factor

β Relative bubble length parameter in a slug flow

δ Liquid film thickness in flow model(m,ft)

$\lambda_L$ No slip liquid holdup

$\mu_G$ Gas viscosity, (Pa.s, cp)

$\mu_L$ Liquid viscosity, (Pa.s, cp)

| | |
|---|---|
| $\mu_M$ | Mixture viscosity, (Pa.s, cp) |
| θ | Inclination angle from horizontal |
| $\rho_G$ | Gas density, (kg/m3,ppg) |
| $\rho_L$ | Liquid density (kg/m3,ppg) |
| $\rho_M$ | Mixture density, (kg/m3,ppg) |
| $\rho_{ML}$ | Mixture density in liquid slug, (kg/m3,ppg) |
| $\rho_{MTB}$ | Mixture density in Taylor bubble in a slug, (kg/m3,ppg) |
| $v_B$ | Discrete gas bubble rise velocity, (m/s,ft/s) |
| $v_G$ | Gas velocity, (m/s,ft/s) |
| $v_L$ | Liquid velocity, (m/s,ft/s) |
| $v_{L_{LS}}$ | Liquid velocity in liquid slug zone, (m/s,ft/s) |
| $v_{L_{TB}}$ | Taylor bubble velocity in a slug, (m/s,ft/s) |
| $v_n$ | Nozzle velocity, (m/s,ft/s) |
| $v_{SG}$ | Superficial gas velocity (m/s,ft/s) |
| $v_{SL}$ | Superficial liquid velocity (m/s,ft/s) |
| $v_{TB}$ | Taylor bubble rise velocity, (m/s,ft/s) |

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# Chapter 1

## Introduction

The Simultaneous flow of oil, gas and water in vertical pipe is encountered in many engineering installations.In petroleum,chemical process,nuclear engineering and many other chemical industries especially in tubing systems,heat exchange equipments and chemical reactor. The problems associated with simultaneous flow of two or more phases through vertical pipe have been of concern for a long time, (Olufemi et al.,2008).

Over the years,accurate prediction of pressure drop has been of vital importances in vertical multiphase flowing oil wells in order to design an effective production string and optimum production strategy selection.various scienctist and reserachers heve proposed correlations and mechanistic models for this purpose since 1950, most of which widely used in the industry.But even with recent improvements in pressure prediction techniques,most of the models fail to provide the desired accuracy of pressure drop,and further improvement is still needed.

Multiphase flow characteristics such as liquid hold up, mixture density, and flow patterns are predict by using Mechanistic models,where the modelling are know as semi-empirical models.These mechanistic models were generated based on sound theoritical approach,to outperform the existing empirical correlations.The most of these mechanistic models are those of (Ansari et al.,1994)

Slug flow is one of the basic flow patterns that characterize the gas–liquid flow in vertical pipes. It occurs over a wide range of gas and liquid flow rates. The most important characteristic of slug flow is its intermittent nature, which is due to a unique phase distribution. In view of the above phase distribution, the pressure and liquid holdup vary periodically at any given pipe cross-section. In vertical flow, the liquid hold up in the slug for prediction and accurate calculation of the pressure drop, the prediction of the liquid hold body is the main factor which contributes to the pressure drop in the piping system.

## 1.2 Underbalanced Drilling

Underbalanced Drilling (UBD) is the drilling process in which the circulating fluid bottomhole pressure is maintained below the formation flowing pressure. UBD can be achieved by injecting lightened drilling fluid such as gas, mist, foam, and diesel, which will create such low pressure in order not to overcome the formation pressure Many benefits are gained from using UBD operations, such as:

• Increase rate of penetration and bit life

• Minimization or elimination of differential sticking

• Minimization of lost circulation

• Reduced formation damage

• Increased well productivity

In addition, UBD operations have increased in recent years due to the following:

• Depleted reservoirs

• Awareness of skin damage

• Elimination of lost circulation

• Cost of differential sticking

• Environmental benefits

UBD techniques can be categorized into two major categories based on the fluid used, which are:

• Gaseous drilling fluid

• Gasified liquid and liquid drilling fluids

During UBD operations, a complex fluid system occurs both inside the drillstring and the annulus. Two phase flow prediction techniques are used to predict several parameters such as pressure drops (both inside the drillstring and through the annulus), flow patterns, velocities, liquid holdup, and other parameters. In order to achieve this, mechanistic two phase flow models are used.

## 1.3 Research Objective

The objective of this research are :

- To predict the pressure drop of slug flow vertical pipes using mechanistic model.
- To predict the behavior of the flow in the certain pressure in the vertical pipes.

## 1.4 Research scope

The research scope that will comply to achieve the research objectives are divided into two stages :

- Study of mechanistic steady state model using Excel Visual Basic Application (VBA) and FORTRAN 95 computer program.
- To study and predict the pressure drop in vertical pipes and the behaviour of the flow.

# CHAPTER 2

# LITERATURE REVIEW

## 2.1 Introduction

In recent years, mechanistic models were developed based on phenomenological approach which mass and energy conservation is been takes account. The early mechanistic model for the vertical flows, Fernandes et al.(1983) developed the semi mechanistic model to predict the liquid hold up in the slug flow body. Sylvester et. al. (1987) modified semi mechanistic for slug flow model by Fernandes, where the new correlation for the liquid holdup is been introduced.

Hasan and Kabir et al.(1992) developed a model for predicting the two phase flow in annuli upward simultaneous two phase flow In UBD operations, pressure along the wellbore length is affected by the gas and liquid injection flow rate, the flow pattern distribution and the back pressure at the wellhead. With the larger well depth, temperature and pressure in annulus increases constantly which results in the varying gas and liquid superficial velocity and gas void fraction which determines flow pattern distribution and pressure.

Ansari et al.(1994) presented the model for upward vertical two phase flow in pipes. Ansari's model improved prediction accuracy of slug flow by considering two possible conditions of slug flow, the fully developed Taylor bubble slug flow and the developing Taylor bubble slug flow.

Bijleveld et al.(1996) developed the first steady state computer program by using the mechanisitic approach,by using trial and errors to calculate the bottom hole pressure and two phase flow parameters. pattern of flow is being assumed, for the purposed of get an accurately prediction of the differences in flow parameters such as rise velocity of gas bubbles in liquid columns, flow pattern and liquid holdup.

Gomez et al.(1999) developed a comprehensive mechanistic model for predicting the flow parameters in deviated wells. Lage et al.(2000) developed a mechanistic model for predicting upward two phase flow in concentric annulus.

## 2.2 Multiphase Flow Concept

Multiphase flow is a generalisation modelling used in two phase flow where the two phase are not chemically related or where two or more phase are present. The most distinguished aspect of such flow during the simultaneous flow of gas and liquid, is the inconsistency of the distribution of both phases in the vertical pipes.the term flow pattern is used to distinguish such distribution,which depends on the relative magnitude of forces acting on the fluids, Brown et al. (1986).The following terms are defined in order to assist in the multiphase flow calculations.

### 2.2.1 Liquid Holdup

Liquid holdup ( HL) is defined as the fraction of a pipe cross-section or volume that is occupied by the liquid phase,Beggs et al. (1991).The value of $H_L$ ranges from 0(total gas) to 1(total liquid).The prediction of liquid Holdup in the slug flow body for two phase gas-liquid slug flow is important for the accurate calculations of the pressure drop.The liquid holdup is defined by

$$H_L = A_L/A_P \qquad 2.1$$

$A_L$ = pipe area of the liquid occupied by the liquid phase

$A_P$ = Pipe cross-sectional area

The term void fraction or gas holdup is defined as the volume fraction occupied by the gas where

$$\alpha = 1 - H_L \qquad 2.2$$

$\alpha$ = gas void fraction

When two fluids travel at different velocities then the flow is referred to as a slip flow. No slip flow occurs when the fluids travels at the same velocity, Hence the term no slip liquid holdup can be defined as the ratio of the volume of liquid in a pipe element that would exist if the gas and liquid traveled at the same velocity divided by the volume of the pipe element,Beggs et al.(1991).

The no-slip liquid Holdup, $\lambda_L$ is defined as follows:

$$\lambda_L = \frac{q_L}{q_{L+}q_G} \quad 2.3$$

$\lambda_L$ = No slip liquid holdup

$q_L$ = Liquid flow rate

$q_G$ = Gas flow rate

### 2.2.2 Superficial Velocity

Superficial velocity is the velocity that a phase would travel at if it flowed through the total cross sectional area available for flow Beggs et al.(1991) Thus, the liquid and gas superficial velocities are defined by :

$$V_{SL} = Q_L / A_p \quad 2.4$$

$V_{SL}$ = Superficial liquid velocity (m/s,ft/s)

$A_p$ = Pipe element area, (m2,in2)

and

$$V_{sg} = Q_G / A_P \quad 2.5$$

$V_{sg}$ = Superficial gas velocity (m/s,ft/s)

The mixture velocity can be defined as the velocity of the two phases together, as follow :

$$V_M = (Q_L + Q_G) / A_P$$

$$= V_{SL} + V_{sg} \quad 2.6$$

The in-situ velocity is the actual velocity of the phase when the two phases travel together. They can be defined as follows :

$$V_L = V_{SL} / H_L \tag{2.7}$$

and

$$V_G = V_{sg} / H_G = V_{sg} / (1- H_L) \tag{2.8}$$

Weighting factor is introduced when water is exist because of the addition to the liquid and gas,this factor is being used to take care of the slippage that could occur between different liquid phases that exists during drilling(drilling fluid, produced oil and produced water). This factor is defined as follows:

$$f_{\circ} = q_{DF} / q_{DF} + q_{\circ} + q_w \tag{2.9}$$

where

$q_{DF}$ = is the drilling fluid flow rate,

$q_{\circ}$ = inflow oil flow rate, and

$q_w$ = is inflow water flowrate.

### 2.2.3 Two Phase flow pattern

Multiphase flow patterns highly depend on flow rates, pipes geometry, and the fluid properties of the phases. The physical distribution of the phase that varies in the flow medium creates several flow patterns. Furthermore, because of the various pressure and temperature in the pipes it also can contribute to the change of the flow pattern. The major flow pattern that exist in multiphase flow are dispersed bubble,bubble,slug,churn and annular as shown in Figure 2.1.

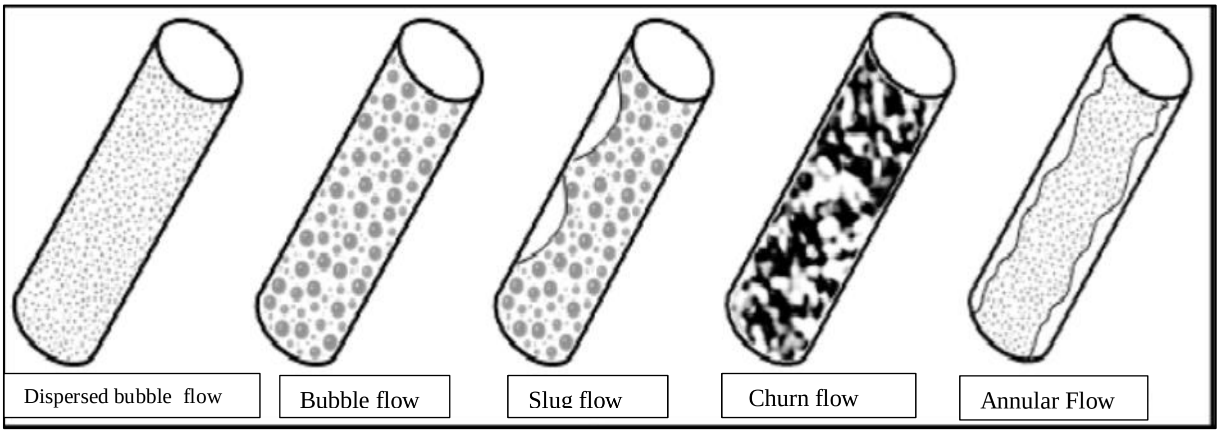


**Figure 2.1** Different flow patterns in Two Phase flow

- **Dispersed bubble flow :** This flow is characterized by gas being distributed in small spherically shaped bubbles in continuous liquid phase.dispersed bubble occurs at low gas flow rates and high liquid rates.in dispersed bubble flow,both phases flow at nearly the same velocity.no slip is seen between the phases and the flow is essentially homogenous.

- **Bubble flow :** This flow characterized by a discontinuous gas phase which is distributed at discrete bubble inside a continuos liquid phase.The discrete gas bubbles tend to slightly deviate from spherical shape and exhibit slippage through the liquid phase due to buoyancy forces.This patern occurs at low to medium superficial velocites.

- **Slug flow :** This flow is characterized by a series of slug units.each unit is composed of Taylor Bubble and plugs of liquid called slugs. Charateristic bullet-shaped bubbles often contains a dispersion of smaller bubbles.A film of liquid exist around the pocket flowing downward relative to the gas bubble.The liquid slug carrying distributed small gas bubbles, bridges the conduit and separates two consecutive gas bubbles.

- **Churn flow :** This flow pattern exist in upward flow only.the shape of the Taylor bubble and the liquid slugs are irregular and random.churn flow can be considered to be a transition between bubbly flow and fully developed slug flow.its characteristics oscillations is an important pattern which covering fairly wide range of gas flow rate,it rgarded as a breaking uf of slug flow with occasional bridging across the tube by the liquid phase at the lower end of the range.While at the higher range of gas flow rates it may be considered a degenerate form of annular flow with the direction of the film flow.

- **Annular flow :** This flow of pattern is characterized by the axial continuity of gas phase in the liquid flowing upward, both as a thin film along the pipe wall and as dispersed droplets in the core. A small amount of liquid is entrained in the light velocity core region. Annular flow occurs at high gas superficial velocities with relatively little liquid present.

Transition boundaries between the various flow patterns can be plotted on a flow pattern map. According to Taitel et al studied, Figure 2.2 shows a typical flow pattern map for downward vertical two phase flow. Figure 2.3 shows the flow pattern map used in the annulus which was developed by Caetano et al.(1992) Both figures are made for certain flow geometries and fluid properties.

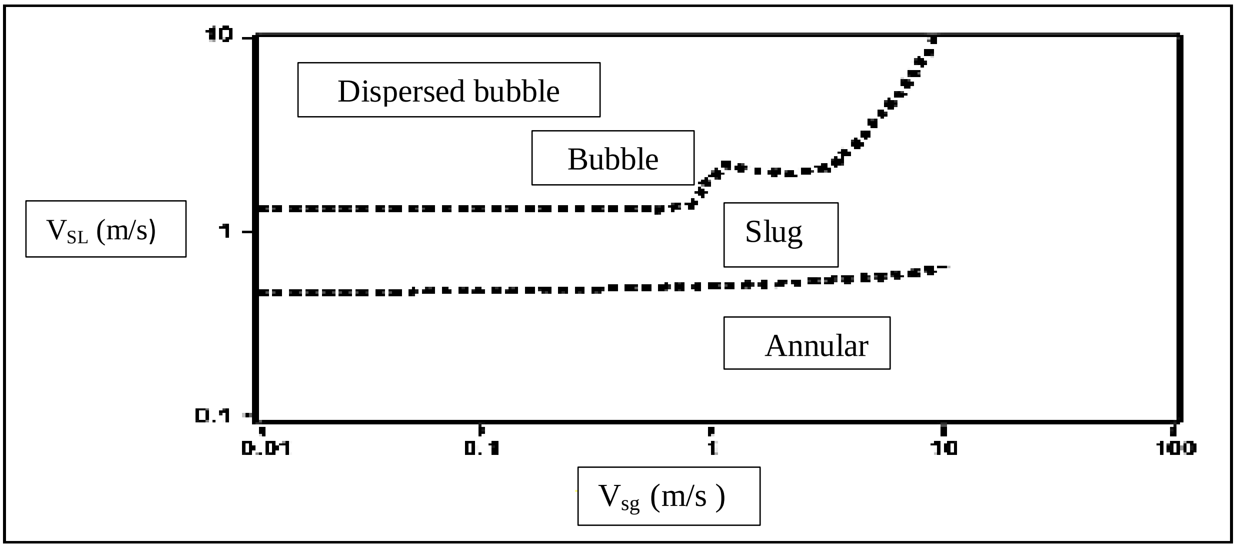


**Figure 2.2:** Flow pattern Map for Downward Two Phase Flow in Pipes

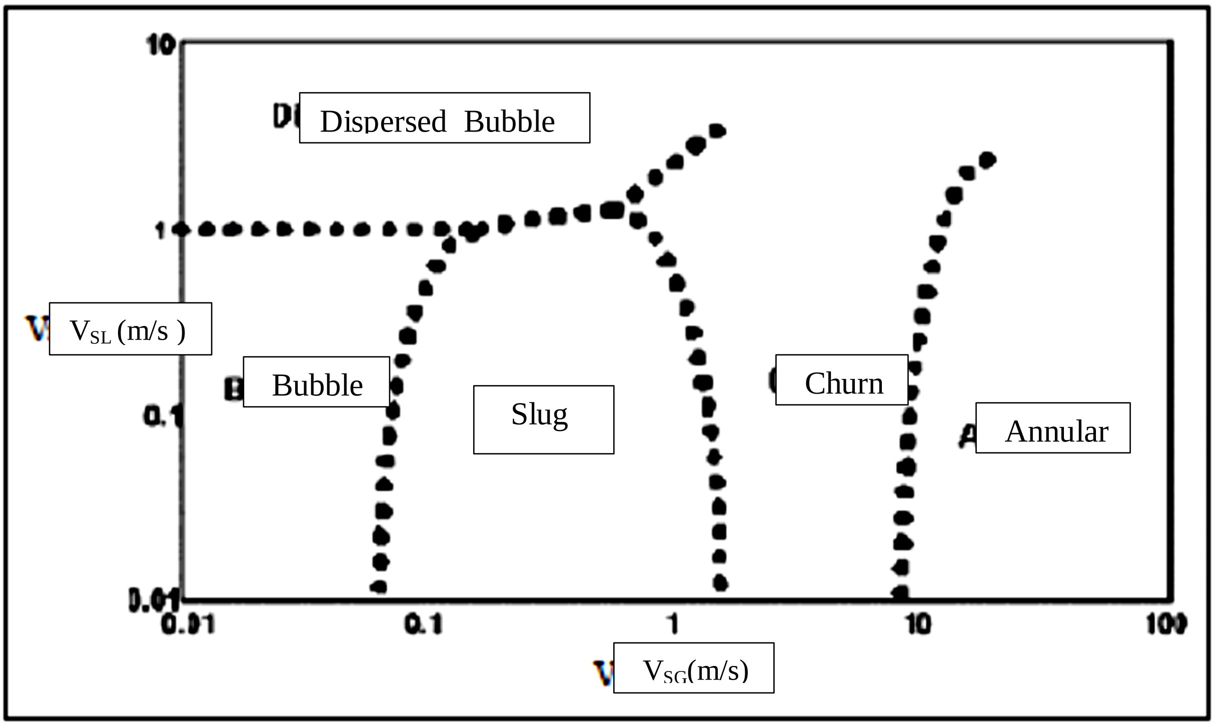


**Figure 2.3 :** Flow Pattern Map For Upward Two Phase Flow in Annulus

### 2.3 Flow Pattern Prediction Models

### 2.3.1 Downward Flow through the Drillstring

#### 2.3.1.1 Bubble to Slug Transition

The transition from bubbly to slug flow occurs because of the bubble resulting from increased collision between bubbles at higher void fraction. In addition, Hasan stated that the same void fraction used for upward flow could be used for the case of downward flow. Hasan observed that this transition occurred at a void fraction of 0.25. Also, the rise velocity is unaffected by pipe inclination angle and in deviated wells, the bubbles 14 prefer to flow near the upper wall of the pipe, causing a higher local void fraction compared with the cross-sectional average value. Hasan and Kabir derived an equation for bubble to slug transition flow for upward flow in deviated wells. Hasan proposes the same equation for a downward flow using a negative terminal rise velocity. Hasan proposed the following expression for transition boundary between bubble and slug flow:

$$V_{SG} = 1 + \frac{C_o n V_{SL}\, x - V_\infty}{(1/\alpha) - C_o} \sin\theta \qquad 2.10$$

Harmathy correlation is used to calculate the terminal rise velocity for upward flow in vertical channels as follows:

$$\underset{\infty}{V} = 1.53 \left[\frac{(\rho_{L-}\rho_G) g\sigma}{\rho_L^2}\right]^{0.252} \qquad 2.11$$

The velocity profile coefficient (*CO*) has been defined by Zuber and Findlay due to the effect of non-uniform flow and concentration distribution across the pipe and the effect of local relative velocity between the two phases. Table 2.1 shows the values for the velocity profile coefficientsfor different inclination angles as given by Alves

**Table 2.1**: Flow Coefficients for Different Inclination Angle Ranges (After Alves)

| Inclination Angle (Degrees) | Co |
|---|---|
| 10-50 | 1.05 |
| 50-60 | 1.15 |
| 60-90 | 1.25 |

In addition, Wallis Wallis, G.B. (1969). has proposed that the effect of single bubble rising in a swarm of bubblescan be introduced by defining a bubble swarm effect (n), thus $H_L^n$ will be taken into consideration. Finally, Perez-Tellez et al proposed the use of the combined effect of the bubble swarm effect (n) and the velocity profile coefficient (*CO*) and introduced the following expression for the bubble slug transition.

$$V_{SL} - C_{\circ} V_m = \pi V_\infty H_L^n \qquad 2.12$$

Applying Equation 2.11 to Hasan approach in order to find the criteria from bubble to slug yields the following equation

$$V_{SL} = \frac{\left( {}^{1}/_{\propto} - C_o \right) V_{SG} / \sin\theta + V_\infty H_L^n}{C_o} \qquad 2.13$$

with a gas void fraction $\alpha = 0.25$.

### 2.3.1.2 Bubble or Slug to Dispersed Bubble Transition

The model which was created by Taitel et al where based on the maximum bubble diameter under highly turbulent conditions could be used to find the relationship between phase velocities, pipe diameters, and fluid properties which applicable for flow through vertical flow. The equation 2.14 which developed by Caetano as shown below was recomended by Perez-Tellez in order to calculate the homogenous fanning friction factor, and since the rise velocity for the dispersed bubble flow is very small compared to the local velocities, the no-slip holdup (λL) could be used to calculate $f_F$. Where ID is the inner pipe diameter.

$$V_M^{1.2} (2 f_F^{0.4} / ID)^{0.4} \left[ \frac{1.6\sigma}{(\rho - \rho) g} \right]^{0.5} (\rho_L / \sigma)^{0.6} = 0.725 + 4.1 (V_{SG} / V_M)^{0.5} \qquad 2.14$$

### 2.3.2 Upward Flow through the Annuli

Taitel et al.(1980) proposed the method for predicting flow pattern, in addition to his model and coupling it with the bubble swarm effect and the velocity swarm coefficient. The flow patterns used were shown in Figure 2.3 where the transition boundaries will be calculated based on different flow geometry and properties.

#### 2.3.2.1 Bubble to Slug Transition

During bubble flow, discrete bubbles rise with the occasional appearance of a Taylor bubble. The discrete bubble rise velocity was defined in Equation 2.11. The presence of an inner tube tends to make the Taylor bubble sharper, causing an increase in the Taylor bubble rise velocity. As a result, Equation 2.15 was developed where the outer tube diameter should be used with the diameter ratio (OD/ID) to get the following expression for the Taylor bubble rise velocity in inclined annulus.

$$V_{TB} = (0.345+0.1*(OD/ID))\sqrt{\sin\theta}\,(1+\cos\theta)^{1.2}\sqrt{gID\frac{\rho L-\rho G}{\rho L}} \qquad 2.15$$

where

OD : Outside pipe diameter

ID : Inner casing diameter

g : Gravity acceleration

$\rho L$: Liquid density

$\rho G$: Gas density

Hasan and Kabir stated that the presence of an inner tube does not appear to influence the bubble concentration profile (*CO*) and thus the following expression could be used :

$$V_{SL} = ((4 - C_O)\,V_{SG})/\sin\theta\,) - V_{\infty} \qquad 2.16$$

where

$C_O$ = Velocity profile coefficient for bubbly flow

$\theta$ = Inclination angle from horizontal

$V_{\infty}$ = Discrete gas bubble rise velocity, (m/s,ft/s)

### 2.3.2.2 Bubble or Slug to dispersed bubble transition

The flow transition from bubble or slug to dispersed bubble been defined by Equation 2.14. The hydraulic diameter (Dh) is substituted for the pipe inside diameter (ID). The hydraulic diameter of the casing-tubing annulus is given by:

$$Dh = ID - OD \qquad 2.17$$

where

ID = internal casing diameter

OD = is the outside pipe diameter.

### 2.3.2.3 Dispersed bubble to slug flow transition

Taitel et al. determined that the maximum allowable gas void fraction under bubble flow condition is 0.52. Higher values will convert the flow to slug, hence the transition boundary could be equated as follows

$$v_{SL} = 0.923\, v_{SG} \qquad 2.18$$

### 2.3.2.4 Slug to churn transition

Tengesdal et al. has developed a transition from slug to churn flow in an annulus. They stated that the slug structure will be completely destroyed and churn flow will occur if the gas void fraction equals 0.78. Thus churn flow will occur. The transition from slug flow to churn flow can thus be represented by :

$$v_{SL} = 0.0684\, v_{SG} - 0.292\sqrt{gD_{ep}} \qquad 2.19$$

where Dep is the equi-periphery diameter defined as follow

$$Dep = ID + OD \qquad 2.20$$

where

ID = is the inner casing diameter

OD = is the outer pipe diameter.